NONLINEAR MODEL OF SHELLS WITH NONDEFORMABLE TRANSVERSE FIBERS

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Under arbitrary rotations and deformations subject to the condition of inextensibility of the transverse fibers, the three-dimensional nonlinear problem of shell deformation is reduced to a two-dimensional problem of the deformation of its basis surface.

The formulated two-dimensional problem requires the insertion of five contour boundary conditions. The nonlinear model it describes for a deformable surface differs from the Cosserat model by the absence of transverse components for the internal moment tensor.

The notation used in [1] is conserved in the elucidation of the material. The Boldface capitals take on the values 1, 2, and 3, while those which are not bold take on the values 1 and 2.

1. Shells as a Three-Dimensional Membrane Continuum

Let the continuum forming the shell be a membrane, and occupy a three-dimensional domain (volume) B with boundary (surface) C in the initial instant (prior to deformation). This domain is parametrized by Lagrange coordinates t_N with basis $\mathbf{A}_{(M)}(t_N)$.

Continuous transformation of the initial basis into the instantaneous basis $\mathbf{A}_{\{M\}}(t_N)$ occurs during shell deformation (the possible time dependence is assumed but not mentioned explicitly). The rigid rotation of the initial basis, which converts it into a rotated basis $\mathbf{A}_{[M]}(t_N)$, is extracted from the complete transformation.

The transformation of the rigid rotation is expressed in terms of the vector field of rotations $\mathbf{V}(t_N)$ by the mutually reciprocal Rodriguez formulas

$$\mathbf{A}_{[N]} = \mathbf{A}_{(N)} + (1/F)\mathbf{V}\times(\mathbf{A}_{(N)} + (1/2)\mathbf{V}\times\mathbf{A}_{(N)}),$$
$$\mathbf{A}_{(N)} = \mathbf{A}_{[N]} - (1/F)\mathbf{V}\times(\mathbf{A}_{[N]} - (1/2)\mathbf{V}\times\mathbf{A}_{[N]}),\quad F = 1+(1/4)\mathbf{V}\cdot\mathbf{V} \tag{1.1}$$

(the author made a misprint in the corresponding formulas in [1]).

The field of rotations conserves the initial metric of the shell, performing only flexures of the lines and surfaces immersed therein. The tensor field $\mathbf{V}_{[M]} = (1/F)(\nabla_M \mathbf{V} + (1/2)\mathbf{V} \times \nabla_M \mathbf{V}) = \mathrm{V}_{[MN]}\,\mathbf{A}^{[N]}$ (∇_M is the partial differentiation operator with respect to t_M) is a measure of these flexures.

The covariant derivatives of the vectors of the rotated basis $\nabla_{(M)}\mathbf{A}_{[N]}=\mathbf{V}_{[M]}\times\mathbf{A}_{[N]}$ ($\nabla_{(M)}$ is the operator of covariant differentiation with respect to t_M in the initial basis) are determined in terms of the flexure tensor.

Transformation of the rotated basis into an instantaneous basis generates a tensor deformation field

$$\mathbf{U}_{[M]} = \mathbf{A}_{\{M\}} - \mathbf{A}_{[M]} = U_{[MN]}\,\mathbf{A}^{[N]}. \tag{1.2}$$

If $\mathbf{U}(t_N)$ is a certain displacement field, then by definition the equality

$$\mathbf{A}_{\{M\}} = \mathbf{A}_{(M)} + \nabla_M\mathbf{U} \tag{1.3}$$

is valid, and

$$\mathbf{U}_{[M]} = \nabla_M\mathbf{U} + \mathbf{A}_{(M)} - \mathbf{A}_{[M]}, \tag{1.4}$$

follows from (1.2), which in combination with (1.1) determines the deformation field in terms of the field of displacements and rotations.

The following variational rules hold for the kinematic fields of the shell (∇_0 is the variation operation):

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$$\nabla_0 A_{[M]} = V_0 \times A_{[M]}, \quad \nabla_M V_0 = \nabla_0 V_{[MN]} A^{[N]}, \quad \nabla_M U_0 - V_0 \times A_{(M)} = \nabla_0 U_{[MN]} A^{[N]},$$
$$V_0 = (1/F)(\nabla_0 V + (1/2) V \times \nabla_0 V), \quad U_0 = \nabla_0 U.$$

The densities, determined in the domain B and referred to its initial metric, for the fields of volume forces $F(t_N)$ and stresses $X^{(M)}(t_N)$ are subject to local (static or dynamic) equations

$$\nabla_{(M)} X^{(M)} + F = 0, \quad A_{(M)} \times X^{(M)} = 0. \tag{1.5}$$

The second of these equations assures the symmetry of the two-basis stress tensor $X^{(MN)} = X^{(M)} \cdot A^{(N)}$, and discloses the asymmetry of the two-basis tensor $X^{(MN]} = X^{(M)} \cdot A^{[N]}$.

The condition

$$[(A_{(\nu)} \cdot A_{(M)}) X^{(M)} - P] \cdot U_0 = 0 \tag{1.6}$$

is satisfied on the boundary of the domain, where $P(t_N)$ is the density defined on the surface C and referred to its initial metric for the surface force field, and $A_{(\nu)}(t_N)$ is the field of normals to the surface C.

The density $W_0(t_N)$ of the shell virtual strain energy is determined by any of the equalities $W_0 = X^{(MN]} \nabla_0 U_{[MN]} = X^{(MN)} \nabla_0 U_{(MN)}$. Here $U_{(MN)}(t_N)$ is the Green symmetric deformation tensor that is independent of the field of rotations:

$$2U_{(MN)} = A_{(M)} \cdot A_{(N)} - A_{(M)} \cdot A_{(N)} = A_{(N)} \cdot \nabla_M U + A_{(M)} \cdot \nabla_N U + \nabla_M U \cdot \nabla_N U$$
$$= A_{[N]} \cdot U_{[M]} + A_{[M]} \cdot U_{[N]} + U_{[M]} \cdot U_{[N]}. \tag{1.7}$$

The possibility of determining the virtual strain energy in terms of the symmetric tensors $X^{(MN)}$ and $U_{(MN)}$, independent of the rigid rotation of the basis, means that such a rotation does not participate at all in the determination of the membrane continuum (is latent according to the Cosserat expression). Hence, the rotated basis is an arbitrary basis for the membrane continuum. The existing arbitrariness appears in the fact that the tensor is not generally symmetric. Any fixing of the rotated basis sets up three scalar couplings between the components of the tensor $U_{[MN]}$ and any three independent couplings attached to the tensor $U_{[MN]}$ fix the location of the rotated basis. The most natural couplings are the symmetry conditions for this tensor: $U_{[MN]} = U_{[NM]}$. However, as will be seen from what follows, other conditions for fixing the rotated basis are more effective in constructing the shell model.

For reversible isothermal and adiabatic processes, the shell strain can be determined by the strain energy function $W(U_{(MN)})$ so that $W_0 = \nabla_0 W = X^{(MN)} \nabla_0 U_{(MN)}$. The equations of a nonlinear elastic relation between symmetric stress and strain tensors (governing the equations) hence follow:

$$X^{(MN)} = \partial W / \partial U_{(MN)}. \tag{1.8}$$

Equations for the coupling between the nonsymmetric tensors $X^{(MN]}$ and $U_{[MN]}$ can be obtained as a corollary of these fundamental equations. The simplest method is to use the dependences $X^{(ML]} = X^{(MN)} A_{(N)} \cdot A^{[L]}$ following from the expansions $X^{(M)} = X^{(ML]} A_{[L]} = X^{(MN)} A_{(N)}$, which result in combination with (1.8) in the coupling equations

$$X^{(ML]} = A_{(N)} \cdot A^{[L]} \, \partial W / \partial U_{(MN)}. \tag{1.9}$$

The derivatives of the potential function are assumed to be expressed here in terms of the nonsymmetric tensor $U_{[MN]}$ components by using (1.7).

The governing equations (1.9) close the formulated system of kinematic and force equations for a shell as a three-dimensional membrane continuum.

2. Shells as a Two-Dimensional Membrane Continuum

The shell mass distribution over points generating its (basis) surface give the latter the meaning of a two-dimensional continuum. For the deformation of such a continuum to model the shell deformation, it should be considered couple-stress. The theory of the nonlinear deformation of a two-dimensional couple-stress continuum can be obtained as a corollary of the theory of the three-dimensional couple-stress continuum [1].

Let the initial spatial basis $\mathbf{A}_{(M)}$ be determined on a surface b with the boundaries (contour) c. Since the surface is parametrized by two internal coordinates t_1 and t_2, $\mathbf{A}_{(M)} = \mathbf{a}_{(M)}(t_n)$. By definition $\mathbf{a}_{(m)}(t_n)$ is the basis of the internal coordinate system, and $\mathbf{a}_{(3)}(t_n)$ is the basis vector normal to the surface. Consequently, the vectors of the two-dimensional initial basis $\mathbf{a}_{(M)}$ are interrelated by orthogonality conditions

$$\mathbf{a}_{(m)} \cdot \mathbf{a}_{(3)} = 0. \tag{2.1}$$

The surface deformation transforms the initial into an instantaneous basis $\mathbf{a}_{\{M\}}(t_n)$ (the possible time dependence is assumed, but not mentioned explicitly). The rigid rotation of the initial basis, carrying it over into the rotated basis $\mathbf{a}_{[M]}(t_n)$, is extracted from this complete transformation. Transformation of the rotation is expressed in terms of the vector field of rotations $\mathbf{v}(t_n)$ by the formulas

$$\begin{gathered}\mathbf{a}_{[N]} = \mathbf{a}_{(N)} + (1/f)\mathbf{v} \times (\mathbf{a}_{(N)} + (1/2)\mathbf{v} \times \mathbf{a}_{(N)}), \\ \mathbf{a}_{(N)} = \mathbf{a}_{[N]} - (1/f)\mathbf{v} \times (\mathbf{a}_{[N]} - (1/2)\mathbf{v} \times \mathbf{a}_{[N]}), \ f = 1 + (1/4)\mathbf{v} \cdot \mathbf{v}\end{gathered} \tag{2.2}$$

and is subject to the conditions

$$\begin{gathered}\mathbf{a}_{[M]} \cdot \mathbf{a}_{[N]} = \mathbf{a}_{(M)} \cdot \mathbf{a}_{(N)} = a_{MN}, \\ \mathbf{a}_{[L]} \cdot (\mathbf{a}_{[M]} \times \mathbf{a}_{[N]}) = \mathbf{a}_{(L)} \cdot (\mathbf{a}_{(M)} \times \mathbf{a}_{(N)}) = d_{LMN},\end{gathered}$$

so that a_{MN} is the metric, and d_{LMN} are the discriminant tensors of the initial and rotated bases simultaneously.

Exactly as the three-dimensional bases $\mathbf{A}_{(M)}$, $\mathbf{A}_{[M]}$, and $\mathbf{A}_{\{M\}}$ are degenerate on the surface b into the two-dimensional bases $\mathbf{a}_{(M)}$, $\mathbf{a}_{[M]}$, and $\mathbf{a}_{\{M\}}$, all the three-dimensional fields introduced in [1] degenerate into two-dimensional fields denoted by the appropriate italic letters: $\mathbf{u}(t_n)$ is the displacement field, $\mathbf{v}(t_n)$ is the rotation field, $\mathbf{u}_{[M]}(t_n)$ is the metric deformation field, $\mathbf{v}_{[M]}(t_n)$ is the flexure field, $\mathbf{x}^{(M)}(t_n)$ and $\mathbf{y}^{(M)}(t_n)$ are linear densities of the internal force and moment fields, $\mathbf{p}(t_n)$ and $\mathbf{q}(t_n)$ are linear densities of the contour force and moment fields, $\mathbf{f}(t_n)$ and $\mathbf{g}(t_n)$ are the densities of the surface (including inertial) force and moment fields.

Since the sections of the two-dimensional continuum are lines belonging to the surface b (in particular, coordinate lines), it can perceive the external effects only because of the internal strains and stresses determined on these lines. Consequently the closed two-dimensional continuum should be subjected to the additional constraints

$$\mathbf{u}_{[3]} = 0, \ \mathbf{v}_{[3]} = 0, \ \mathbf{x}^{(3)} = 0, \ \mathbf{y}^{(3)} = 0. \tag{2.3}$$

A corollary of the first constraint is the equality $\mathbf{a}_{\{3\}} = \mathbf{a}_{[3]}$, meaning that during deformation the normal vector performs just rigid rotation by remaining nondeformable. The two remaining vectors of the instantaneous basis do not agree with the corresponding vectors of the rotated basis in the general case. Moreover, if the vectors of the rotated basis are subject to orthogonality conditions of the form (2.1), then these conditions are not satisfied for vectors of the instantaneous basis in the general case. It follows from the sequential chain of equalities $\mathbf{a}_{\{m\}} \cdot \mathbf{a}_{\{3\}} = (\mathbf{a}_{[m]} + \mathbf{u}_{[m]}) \cdot \mathbf{a}_{[3]} = \mathbf{u}_{[m]} \cdot \mathbf{a}_{[3]}$ that the orthogonality conditions of the form (2.1) for the instantaneous basis are satisfied only in the absence of normal components for the vectors

The operations of reducing the dimensionality and subjecting the equations of the three-dimensional couple-stress continuum to the constraints (2.3) [1] result in a closed system of deformation equations for a two-dimensional couple-stress continuum. It is formed by the following groups of equations.

1. Kinematic equations determining the tensorial flexure and deformation fields in terms of the independent vector displacement and rotation fields:

$$\begin{gathered}\mathbf{u}_{[m]} = u_{[mN]}\,\mathbf{a}^{[N]} = \mathbf{a}_{\{m\}} - \mathbf{a}_{[m]} = \nabla_m \mathbf{u} - (1/f)\mathbf{v} \times (\mathbf{a}_{(m)} + (1/2)\mathbf{v} \times \mathbf{a}_{(m)}) \\ = \nabla_m \mathbf{u} - (1/f)\mathbf{v} \times (\mathbf{a}_{[m]} - (1/2)\mathbf{v} \times \mathbf{a}_{[m]}), \ \mathbf{v}_{[m]} = v_{[mN]}\,\mathbf{a}^{[N]} = (1/f)(\nabla_m \mathbf{v} \\ + (1/2)\mathbf{v} \times \nabla_m \mathbf{v}).\end{gathered} \tag{2.4}$$

The continuity conditions (strain compatibility conditions) of a two-dimensional continuum

$$d^{3mn}(\nabla_{(m)}\mathbf{v}_{[n]} - (1/2)\mathbf{v}_{[m]}\times\mathbf{v}_{[n]}) = 0,\quad d^{3mn}(\nabla_{(m)}\mathbf{u}_{[n]} + \mathbf{v}_{[m]}\times\mathbf{a}_{[n]}) = 0$$

are a corollary of (2.4) ($\nabla_{(m)}$ is the operator of covariant differentiation with respect to t_m in the initial basis $a_{(M)}$.

The covariant derivatives of the vectors of the rotated basis are evaluated by the rule $\nabla_{(m)}\mathbf{a}_{[N]} = \mathbf{v}_{[m]}\times\mathbf{a}_{[N]}$ by the equalities to be determined

$$\nabla_{(m)}\mathbf{a}_{[n]} = \nabla_m\mathbf{a}_{[n]} - c^{l}_{\cdot nm}\mathbf{a}_{[l]} + b_{mn}\mathbf{a}_{[3]},\ \nabla_{(m)}\mathbf{a}_{[3]} = \nabla_m\mathbf{a}_{[3]} - b_{mn}\mathbf{a}^{[n]} \tag{2.5}$$

The $c^{l}_{\cdot mn}$ in (2.5) are the Christoffel internal coordinate system of the second kind for a surface, and $b_{mn} = \mathbf{a}_{(n)}\cdot\nabla_m\mathbf{a}_{(3)}$ is the tensor of initial surface curvature.

2. Force (static and dynamic) equations relating the tensor internal force and moment fields:

$$\nabla_{(m)}\mathbf{x}^{(m)} + \mathbf{f} = 0,\ \nabla_{(m)}\mathbf{y}^{(m)} + \mathbf{a}_{(m)}\times\mathbf{x}^{(m)} + \mathbf{g} = 0. \tag{2.6}$$

3. Conditions on the boundary contour c:

$$[(\mathbf{a}_{(\nu)}\cdot\mathbf{a}_{(m)})\mathbf{x}^{(m)} - \mathbf{p}]\cdot\mathbf{u}_0 = 0,\ [(\mathbf{a}_{(\nu)}\cdot\mathbf{a}_{(m)})\mathbf{y}^{(m)} - \mathbf{q}]\cdot\mathbf{v}_0 = 0 \tag{2.7}$$

($\mathbf{a}_{(\nu)}(t_n)$ is the field of unit normals to the contour tangent to the surface b, and $\mathbf{u}(t_n)$ and $\mathbf{v}_0(t_n)$ are the virtual displacement and rotation fields).

4. The expression for the surface density $w_0(t_n)$ of the virtual strain energy and the governing equations for the reversible isothermal and adiabatic strain processes of a two-dimensional continuum:

$$w_0 = x^{(mN)}\nabla_0 u_{[mN]} + y^{(mN)}\nabla_0 \dot{v}_{[mN]}, \tag{2.8}$$

$$w_0 = \nabla_0 w,\ w = w(u_{[mN]}, v_{[mN]}),\ x^{(mN)} = \partial w/\partial u_{[mN]},\ y^{(mN)} = \partial w/\partial v_{[mN]}.$$

The nonlinear model of a two-dimensional couple-stress continuum formed by (2.4) and (2.6)-(2.8) agrees with the Cosserat model presented in [2]. The two-dimensional continuum described by this model is called a Cosserat surface.

In such a formal construction of the model of a two-dimensional couple-stress continuum, its correspondence to the problem of shell deformation as a real three-dimensional body remains undisclosed.

3. Shells as a Three-Dimensional Membrane Continuum with Nondeformable Fibers

The specifics of shells permits the insertion of a spatial coordinate system t_N associated with the basis surface b immersed therein. The parameters t_1 and t_2 are defined as internal coordinates of this surface, while the parameter t_3 is defined as the normal coordinate.

Two initial bases, the three dimensional-basis $\mathbf{A}_{(M)}(t_N)$, defined in the whole volume of the shell, and the two-dimensional basis $\mathbf{a}_{(M)}(t_n)$, defined on the basis surface, are set in correspondence to the coordinate system introduced in this manner. By the definition of the shells, these bases are interrelated by the equalities

$$\mathbf{A}_{(m)} = (a_{mn} + b_{mn}t_3)\mathbf{a}^{(n)},\ \mathbf{A}_{(3)} = \mathbf{a}_{(3)}. \tag{3.1}$$

The shell deformation converts the initial basis into the corresponding instantaneous bases $\mathbf{A}_{\{M\}}(t_N)$ and $\mathbf{a}_{\{M\}}(t_n)$. The rigid rotation generating the rotated bases $\mathbf{A}_{[M]}(t_N)$ and $\mathbf{a}_{[M]}(t_n)$ is extracted from their complete transformation. The arbitrariness allowable here permits expression of the rotation of both bases in terms of a two-dimensional field of rotations $\mathbf{v}(t_n)$ by formulas of the form (1.1) and (2.2). Consequently, constraints of the form (3.1)

$$\mathbf{A}_{[m]} = (a_{mn} + b_{mn}t_3)\mathbf{a}^{[n]},\ \mathbf{A}_{[3]} = \mathbf{a}_{[3]} \tag{3.2}$$

are conserved between the rotated bases.

Moreover, the metric and discriminant tensors of the rotated bases agree with the corresponding tensors of the initial bases.

To match the theory of Cosserat surfaces elucidated in the previous section, the shell deformation is subjected to the kinematic constraint

$$\mathbf{A}_{(3)} = \mathbf{a}_{[3]}, \tag{3.3}$$

which means that the transverse shell fibers are not deformed but only perform rigid rotation.

The chain of equalities $\mathbf{A}_{(3)} = \mathbf{A}_{[3]} = \mathbf{a}_{(3)} = \mathbf{a}_{[3]}$ is a corollary of (3.2) and (3.3).

A linear distribution with respect to the normal coordinate of the field of shell displacements

$$\mathbf{U} = \mathbf{u} + (\mathbf{a}_{[3]} - \mathbf{a}_{(3)})t_3 \tag{3.4}$$

is set up as a result of integrating the equation $\nabla_3 \mathbf{U} = \mathbf{a}_{[3]} - \mathbf{a}_{(3)}$ following from (1.3), (3.1), and (3.3).

Then the shell deformation field

$$\mathbf{U}_{[3]} \equiv 0, \ \mathbf{U}_{[m]} = \mathbf{u}_{[m]} + \mathbf{v}_{[m]} \times \mathbf{a}_{[3]} t_3 \tag{3.5}$$

corresponding to the distribution (3.4) is determined by means of (1.4). Here $\mathbf{u}_{[m]}$ and $\mathbf{v}_{[m]}$ are the tensor strain fields of the basis surface defined by Eqs. (2.4).

The variational equalities

$$\int_B \left[\left(\nabla_{(M)}\mathbf{X}^{(M)} + \mathbf{F}\right)\cdot \mathbf{U}_0 + \left(\mathbf{A}_{(M)}\times\mathbf{X}^{(M)}\right) \cdot \mathbf{V}_0\right] dB = \tag{3.6}$$

$$= \int_b \left[\left(\nabla_{(m)}\mathbf{x}^{(m)} + \mathbf{f}\right)\cdot \mathbf{u}_0 + \left(\nabla_{(m)}\mathbf{y}^{(m)} + \mathbf{a}_{(m)}\times\mathbf{x}^{(m)} + \mathbf{g}\right)\cdot \mathbf{v}_0\right] db = \int_b (\mathbf{f}\cdot\mathbf{u}_0 + \mathbf{g}\cdot\mathbf{v}_0 -$$

$$- w_0)\, db + \int_c (\mathbf{p}\cdot\mathbf{u}_0 + \mathbf{q}\cdot\mathbf{v}_0)\, dc = 0,$$

which have the meaning of the principle of virtual shell displacements, yield two-dimensional force equations of the form (2.6) defined on the basis surface, boundary conditions of the form (2.7) defined on its contour, and an expression for the surface density of the virtual strain energy of the form

$$w_0 \equiv \int W_0 dt_3 = x^{(mN]}\nabla_0 u_{[mN]} + y^{(mn]}\nabla_0 v_{[mn]}. \tag{3.7}$$

Moreover, Eqs. (3.6) disclose the meaning of the two-dimensional force fields as three-dimensional fields averaged over the shell thickness:

$$\mathbf{x}^{(m)} = \frac{1}{j}\int \mathbf{X}^{(m)} J\, dt_3, \ \mathbf{y}^{(m)} = \mathbf{a}_{[3]}\,\frac{1}{j}\int \mathbf{X}^{(m)} J t_3 dt_3, \ \mathbf{p} = \frac{1}{j}\int \mathbf{P} J dt_3, \tag{3.8}$$

$$\mathbf{q} = \mathbf{a}_{[3]}\times\frac{1}{j}\int \mathbf{P} J t_3 dt_3, \ \mathbf{f} = \frac{1}{j}\int\left[\mathbf{F}J + \nabla_3\left(\mathbf{X}^{(3)}J\right)\right] dt_3, \ \mathbf{g} = \mathbf{a}_{[3]}\times\frac{1}{j}\int\left[\mathbf{F}J t_3 + \nabla_3\left(\mathbf{X}^{(3)}J t_3\right)\right] dt_3$$

($j = j(t_n)$ is the Jacobian of the basis $\mathbf{a}_{(N)}$, $J = J(t_N)$ is the Jacobian of the basis $\mathbf{A}_{(N)}$, and for simplicity the limits of integration in the variable t_3 are omitted).

For the known coupling equations (1.9) between the three-dimensional stress $\mathbf{X}^{(M)}$ and strain $\mathbf{U}_{[M]}$ fields, the upper Eqs. (3.8) also have the meaning of governing equations of a two-dimensional continuum. In fact, Eqs. (3.5) express the three-dimensional strain field $\mathbf{U}_{[M]}$ in terms of the two-dimensional fields $\mathbf{u}_{[M]}$ and $\mathbf{v}_{[M]}$. The three-dimensional stress field $\mathbf{X}^{(M)}$ and the two-dimensional force field $\mathbf{x}^{(m)}$, $\mathbf{y}^{(m)}$ are expressed in terms of these two-dimensional fields by means of (1.9) and (3.8).

The equalities following from the definition of the vectors $\mathbf{y}^{(m)}$, $\mathbf{p}$, and $\mathbf{g}$

$$\mathbf{y}^{(m)}\cdot\mathbf{a}_{[3]} = 0, \ \mathbf{q}\cdot\mathbf{a}_{[3]} = 0, \ \mathbf{g}\cdot\mathbf{a}_{[3]} = 0 \tag{3.9}$$

indicate that the vector internal and external moment fields in the rotated basis are two-component fields.

Equations (3.9) are additional conditions that distinguish the closed system of equations of basis surface deformation immersed in the three-dimensional membrane continuum with the kinematic constraint (3.3) from the model of the Cosserat deformable surface. In the latter model, besides the ordinary "shell" moments that are a result of averaging the stresses over the shell thickness, there are still normal moments $y^{(m3]} = \mathbf{y}^{(m)}\cdot\mathbf{a}^{[3]}$ generated by the local couple-stress of the two-dimensional continuum. These moments perform work on the increments of the normal components $v_{[m3]} = \mathbf{v}_{[m]}\cdot\mathbf{a}_{[3]}$ of the flexure tensor.

The differential order of the system of equations of the deformable Cosserat surface formulated in the preceding section is twelve, and boundary conditions (2.7) are formulated for it by six scalar equations. Subjecting this system of constraints (3.9) reduces its order to ten, and reduces the number of scalar boundary conditions to five. Precisely such a number of resultant force factors occurs in reducing the forces distributed over the boundary section of the shell to the basis surface.

Summarizing, it can be asserted that the kinematics of the basis surface in a continuum-shell with constraint (3.3) is identical to the kinematics of the Cosserat surface, while the statics (dynamics) is distinct because of additional constraints (3.9). Meanwhile, the model of the shell as a three-dimensional membrane continuum subjected to kinematic constraint (3.3) includes not only the system of two-dimensional equations describing the deformation of the basis surface but also the three-dimensional equations (3.5), (3.4), (1.9), (1.6), and (1.5) which permit construction of the three-dimensional problem for a shell by means of the solution of the two-dimensional problem for a surface. Consequently, the model of a shell as a three-dimensional membrane continuum with nondeformable transverse fibers is richer in content than the model of a two-dimensional couple-stress continuum.

Subjecting the model constructed here for a shell to the additional kinematic constraint $\mathbf{u}_{[m]}\cdot\mathbf{a}_{[3]}=0$ transforms it into a nonlinear Kirchhoff model [3].

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